Solvency II yield curves

EIPOA, May 5, 2011

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Overview

- Presentation
- The QIS 5 yield curve
- The basic risk free curve
- The choice of UFR
- The illiquidity premium

Presentation

- Partner, Head of Research at Scanrate Financial Systems, Aarhus, Denmark
- Have worked with fixed income modeling since 1984
- The presentation reflects my personal view on yield curve methods
- Relatively new to the Solvency 2 discussion
- However, the framework is extremely well documented on the EIOPA web site!

About Scanrate

- Founded 1986, spin-off from fixed income research at Aarhus University, Denmark.
- Staff: 20 employees, mostly academic, finance, mathematical economics, information technology. 3 Ph.D. sponsorships.
- Product: The fixed income system RIO
- Clients: Danish banks, pension funds, mutual funds
- Main services:
  - Scanrate Prepayment and Valuation service
  - Fair value pricing of bonds (Service provider for SIX Telekurs)
  - Scanrate Danish mortgage bond model (with Thomson Reuters)
  - Risk management, consultancy, seminars and support
- More information on www.scanrate.dk

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The QIS 5 yield curve

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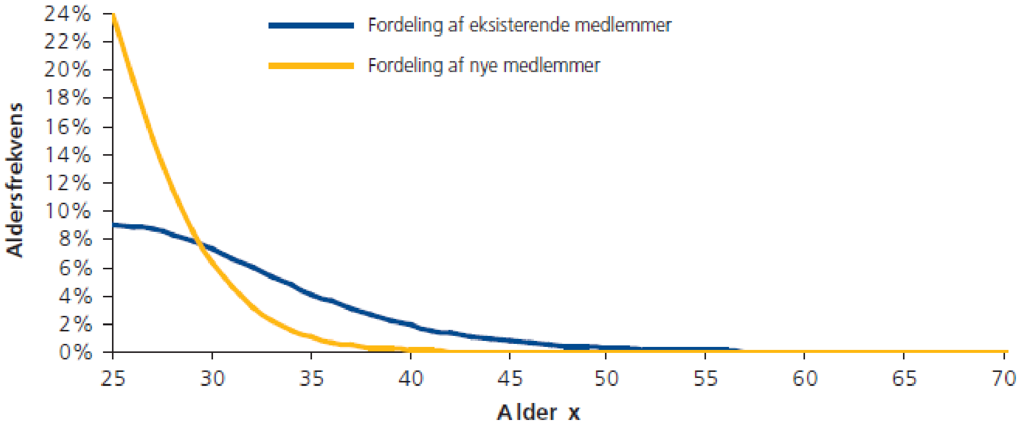
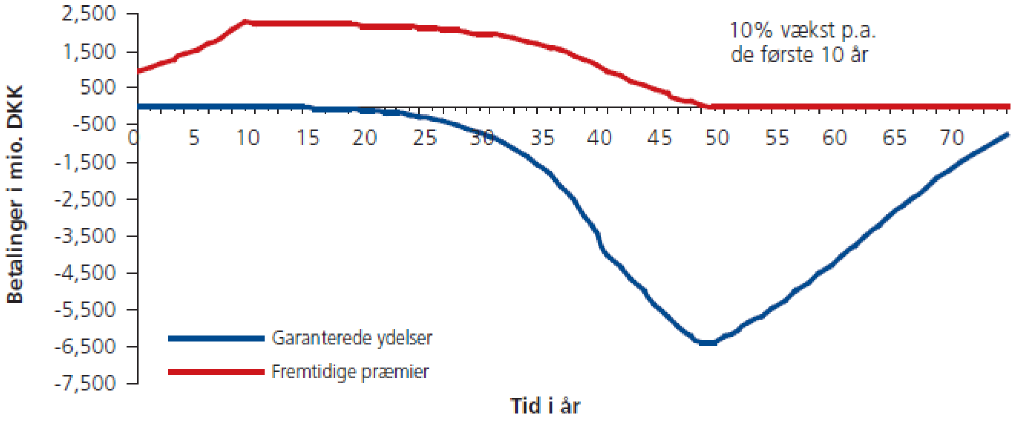
Life long guarantees – the yield curve problem

- Example of pension fund cash flow:
- Fig 1: Age distribution of old (blue) and new participants
- Fig 2: Premiums (red) and liabilities (blue) assuming 10% growth for 10 years, life long guarantees at 2%
- Liabilities are long dated and sensitive to long term yields
- Long term yields are not observable or traded in most markets
- The Solvency 2 long term yields have to rely on assumptions with no direct market basis
- Such inputs or assumptions will affect the solvency capital levels in institutions





Source: Mikkel Svenstrup and Lars Winkel

The Solvency 2 / QIS 5 yield curve

- The basic risk free term structure is based on zero coupon swap yields
- Forward rates beyond 90 years equal the unconditional ultimate forward rate (UFR)
- Extrapolation towards UFR starts at the last liquid swap maturity (the entry point)
- Extrapolation and intrapolation between zero yields uses the Smith-Wilson method
- An illiquidity premium is determined for each currency based on bond indices
- The illiquidity premium is added to the *spot* curve before the entry point. Beyond the entry point the premium is set to zero

The basic risk free curve

QIS 5 choice of basic risk-free curve

- QIS 5 uses zero coupon yield curves provided by the CFO/CRO Forum
- The curves are derived from Libor/Euribor based swap market quotes using Barrie & Hibbert’s regression spline approach (<1 bp average absolute deviation from quotes)
- The last liquid maturity is determined for each currency (Entry point for extrapolation)
- For QIS 5 the zero coupon yields are fitted exactly and the curve extrapolated to 135 years using the SW method
- For most curves the speed of reversion parameter alpha in the SW formula equals 0.1, but the parameter may be increased to ensure that the forward rates reach UFR before 90 years

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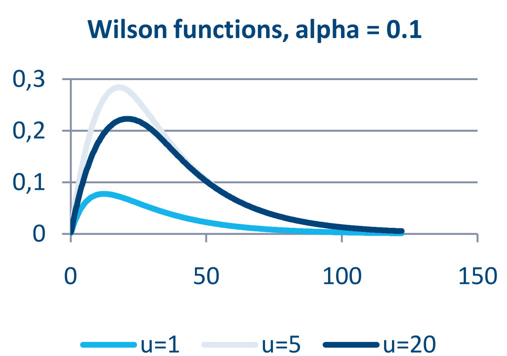
The Smith-Wilson method

- QIS 5 uses a yield curve method proposed by Smith and Wilson(2001)
- The SW curve models the discount function directly as a fixed rate discount plus a weighted sum of smooth functions, W(t,u_k) each of which starts at zero, has a maximum at maturity u_k and converges back to zero for t going towards infinity



Given:

- long term forward rate, *UFR*,
- α, measuring the speed of convergence to *UFR*
- maturity points, u_k, $k = 1, \dots, K$,

Smith-Wilson assumes that the discount factor, $\mathrm{P(t)}$, at time *t* is determined by

$$P(t) = e^{-UFR \cdot t} + \sum_{k=1}^{K} \zeta_k \cdot W(t, u_k)$$

with the symmetric function, $W(t, u_k)$, defined as

$$\begin{aligned} W(t, u_k) = e^{-UFR \cdot (t+u_j)} & \\ & \cdot \left\{ \alpha \cdot \min\left(t, u_j\right) - 0.5 \cdot e^{-\alpha \cdot \min(t, u_j)} \right. \\ & \left. \cdot \left(e^{\alpha \cdot \min(t, u_j)} - e^{-\alpha \cdot \min(t, u_j)} \right) \right\} \end{aligned}$$

ζ_k is a set of *K* parameters to be determined from market data

Our implementation of SW

- Scanrate estimates government, swap, rating, domicile, sector and individual issuer curves on a daily basis for 12 different currencies. More than 1500 individual yield curves updated several times a day.
- Corporate curves are estimated as spread curves to swap curves
- Our standard method is to model the yield curve as a smoothing cubic spline transformed to ensure stable long term yields
- For this presentation we have implemented the SW method in the general case with K maturity points and N instruments (zero yields, bonds or swaps)
- If N = K an exact fit to quotes can be found by the solution of K linear equations. No need to convert swap quotes to zero yields before SW bootstrap
- If N > K the parameters may be found by OLS or GLS regression as shown by McCulloch(1971). An exact fit is not expected.
- Smoothing of yield curve may be done by removing maturity points or penalizing curvature at the cost of an exact fit

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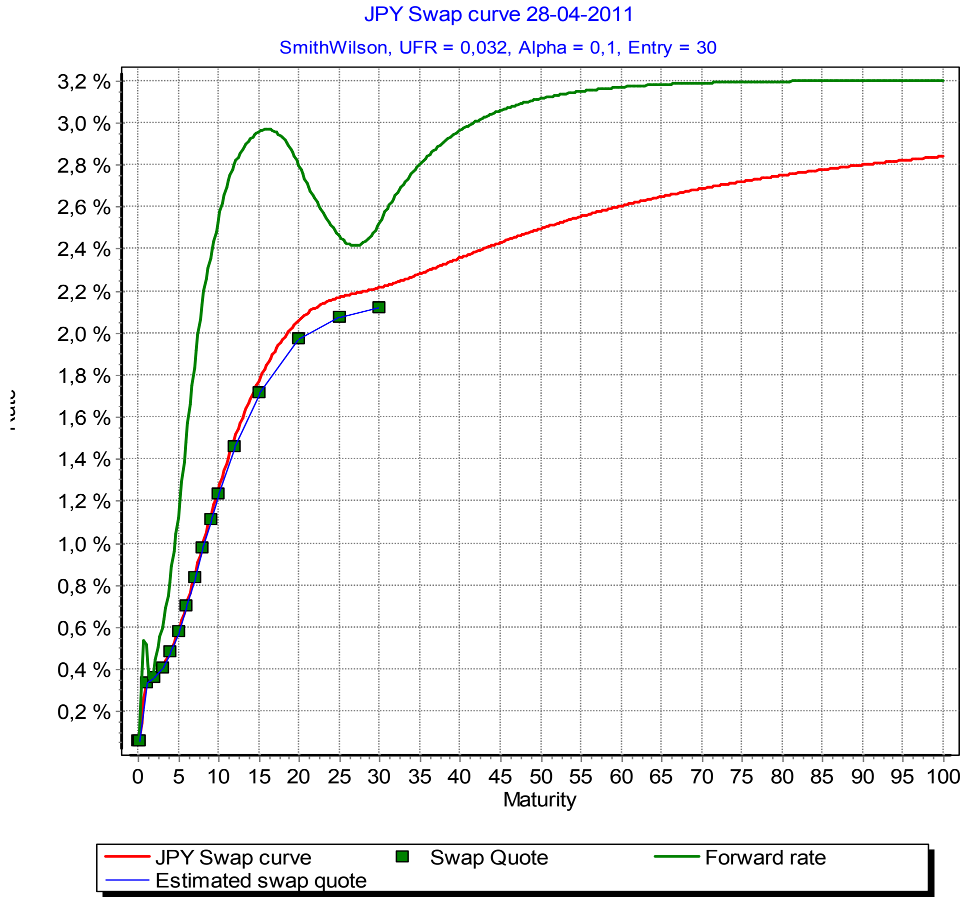
SW yield curve examples

- The next slides show examples of zero coupon yield curves estimated using the SW method
- UFR, alpha and the entry points are set in accordance with QIS 5 (except for a choice of continuously compounding)
- The samples include deposit rates below 1Y
- Swap payments have been calculated according to conventions of each specific market
- Par swap rates (green square) are shown with annual compounding
- The blue curve shows the par swap rates calculated from the from the yield curve
- Swap rates beyond the entry point will not be fitted exactly



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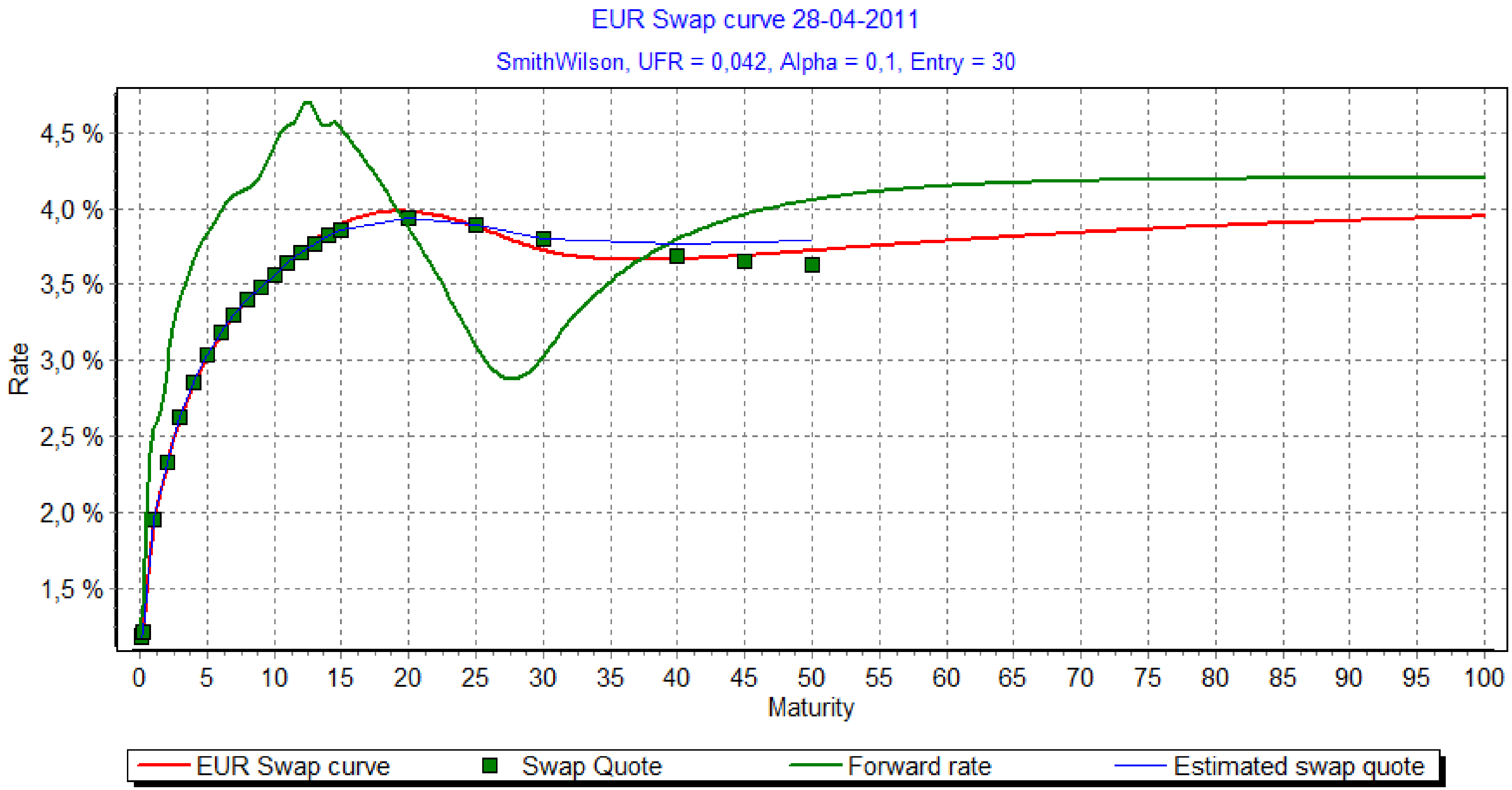
EUR Swap curve



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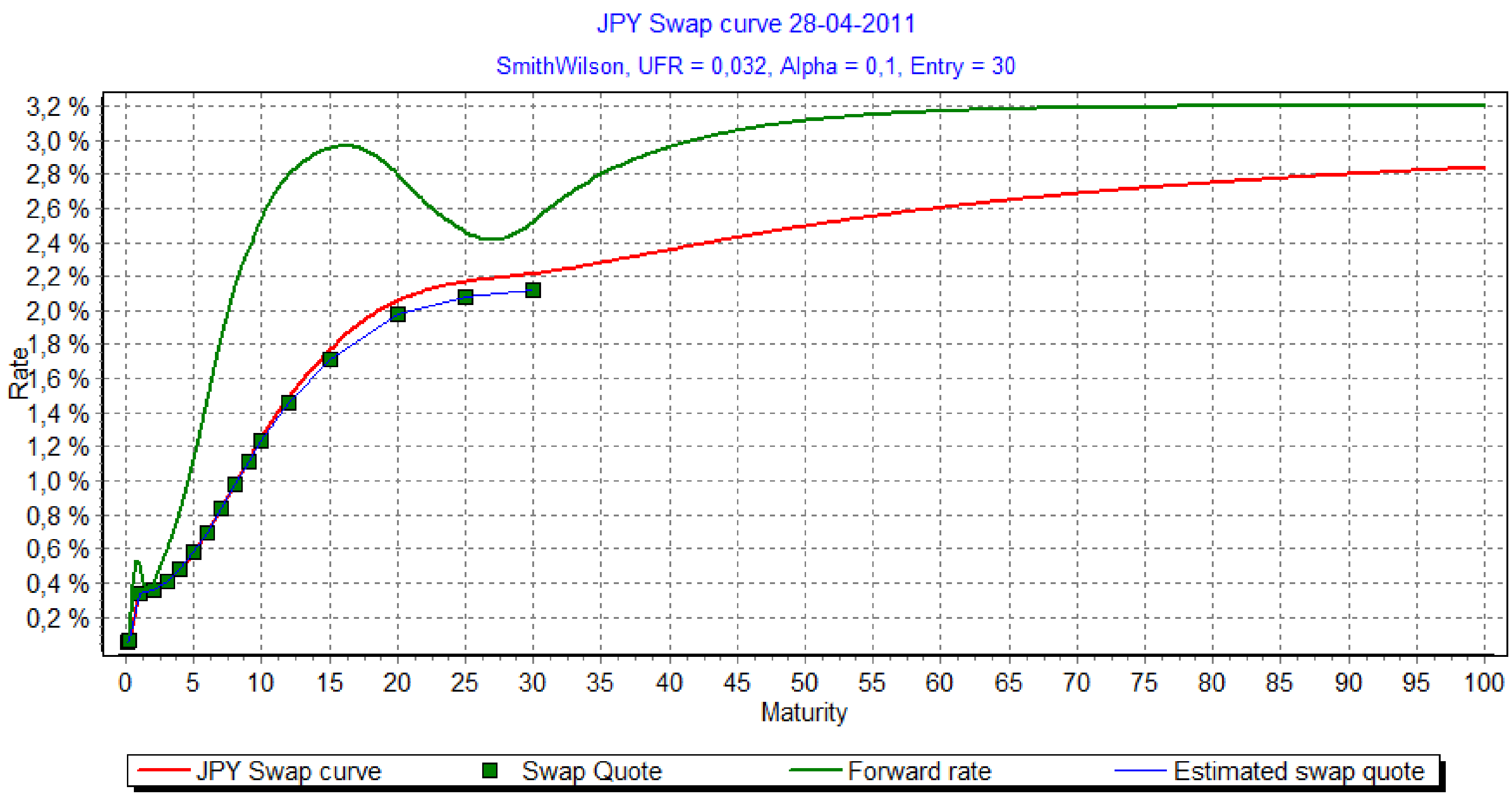
JPY swap curve



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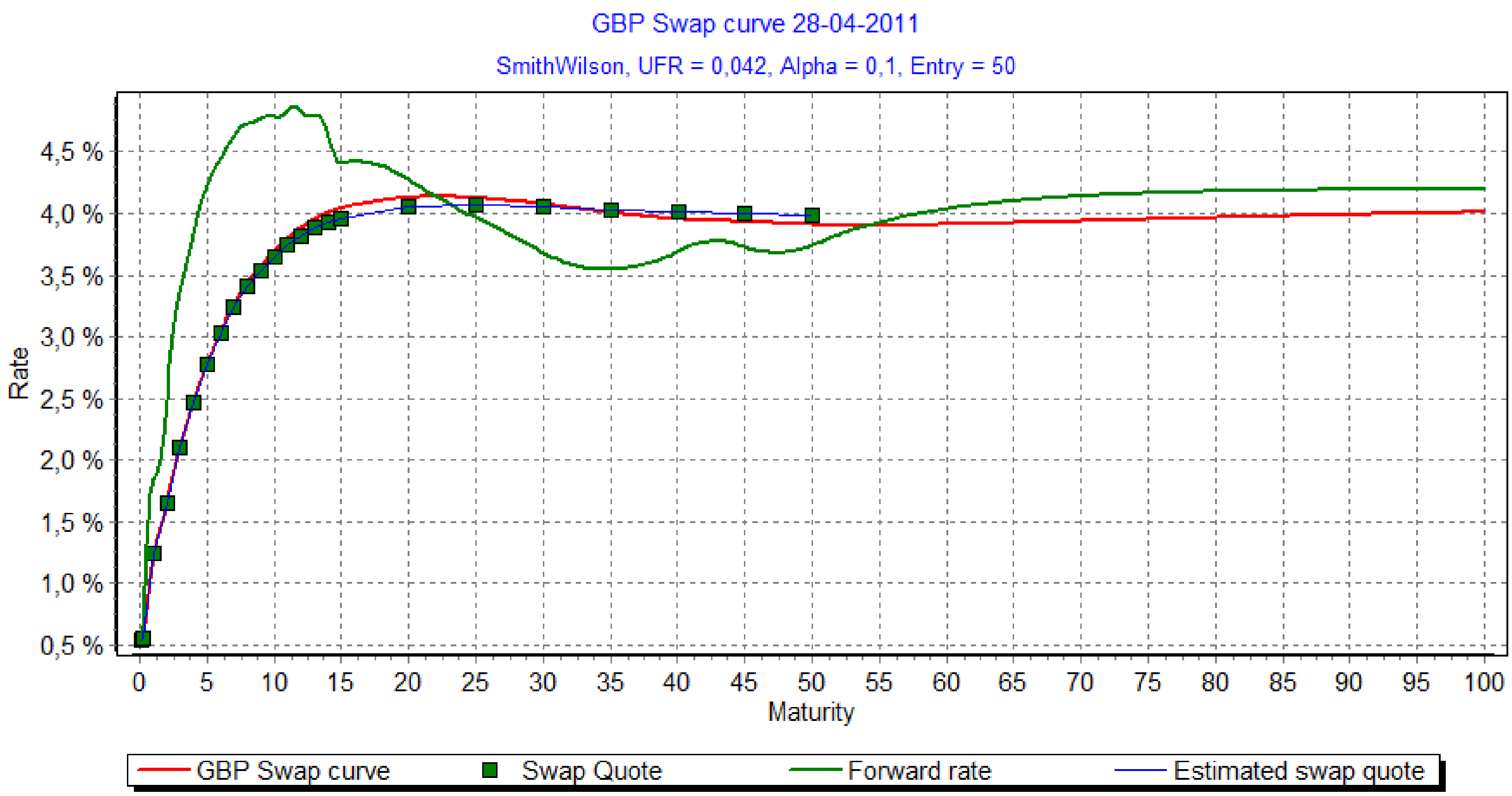
GBP Swap curve



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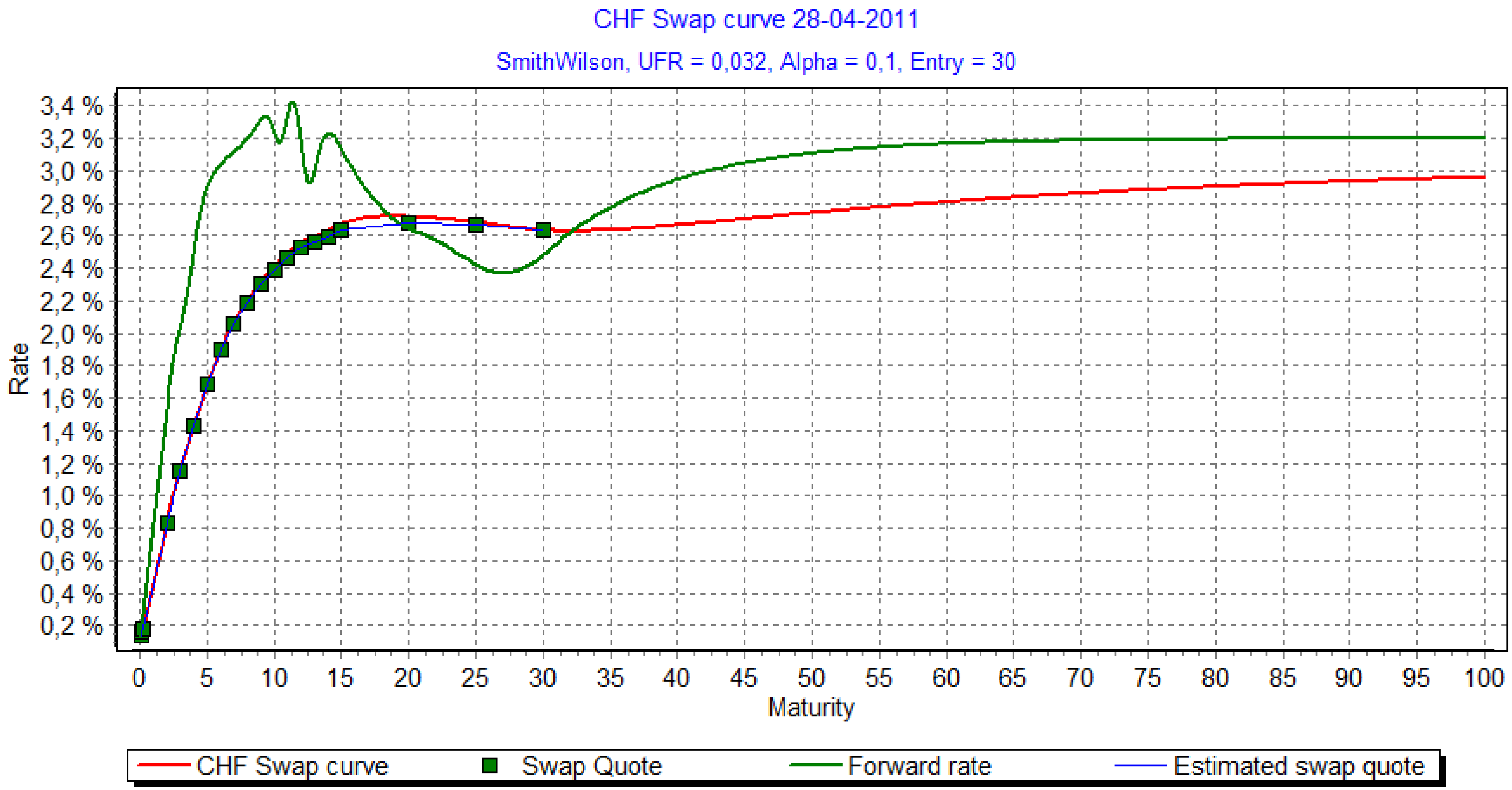
CHF curve



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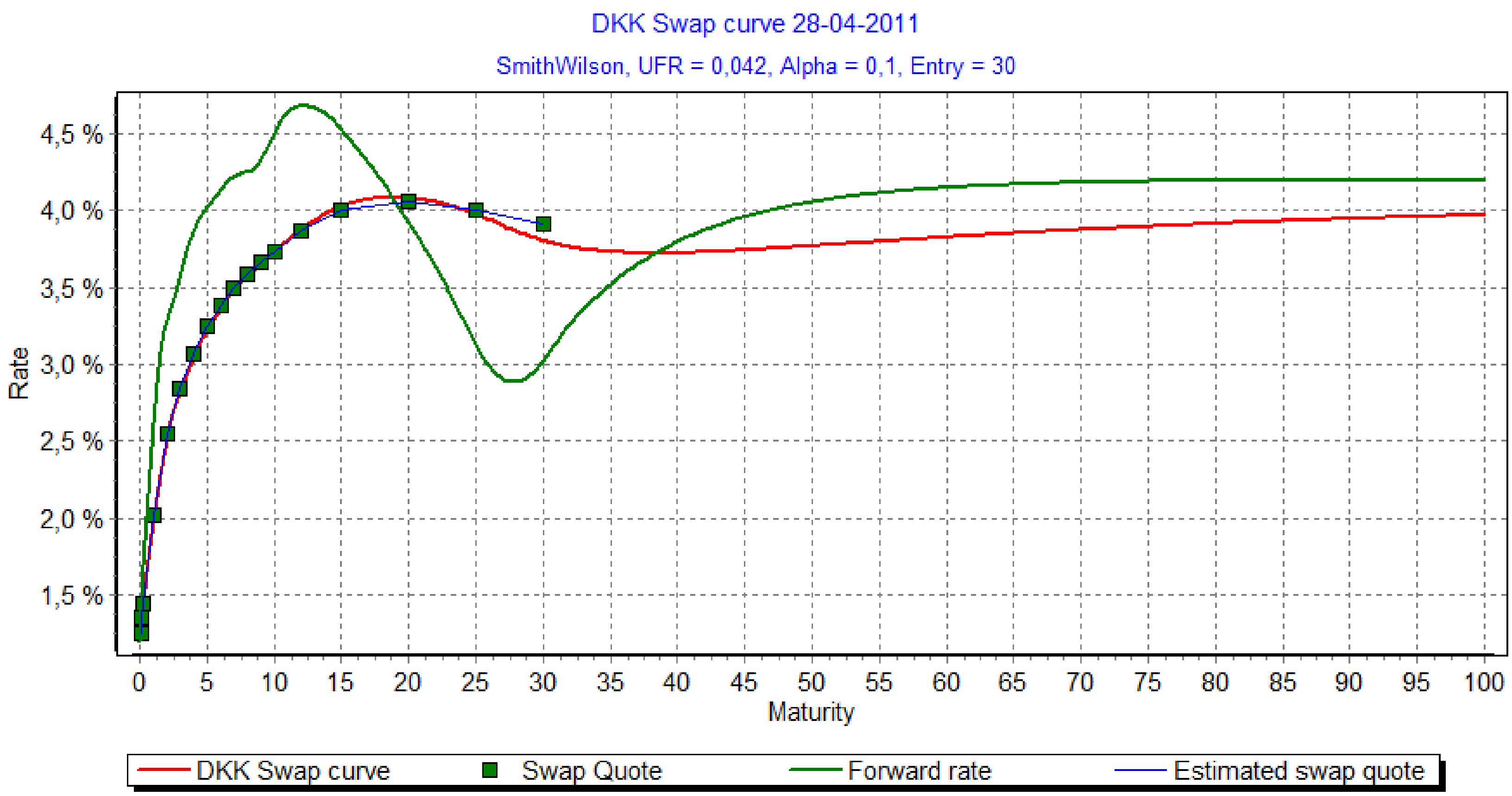
DKK swap curve



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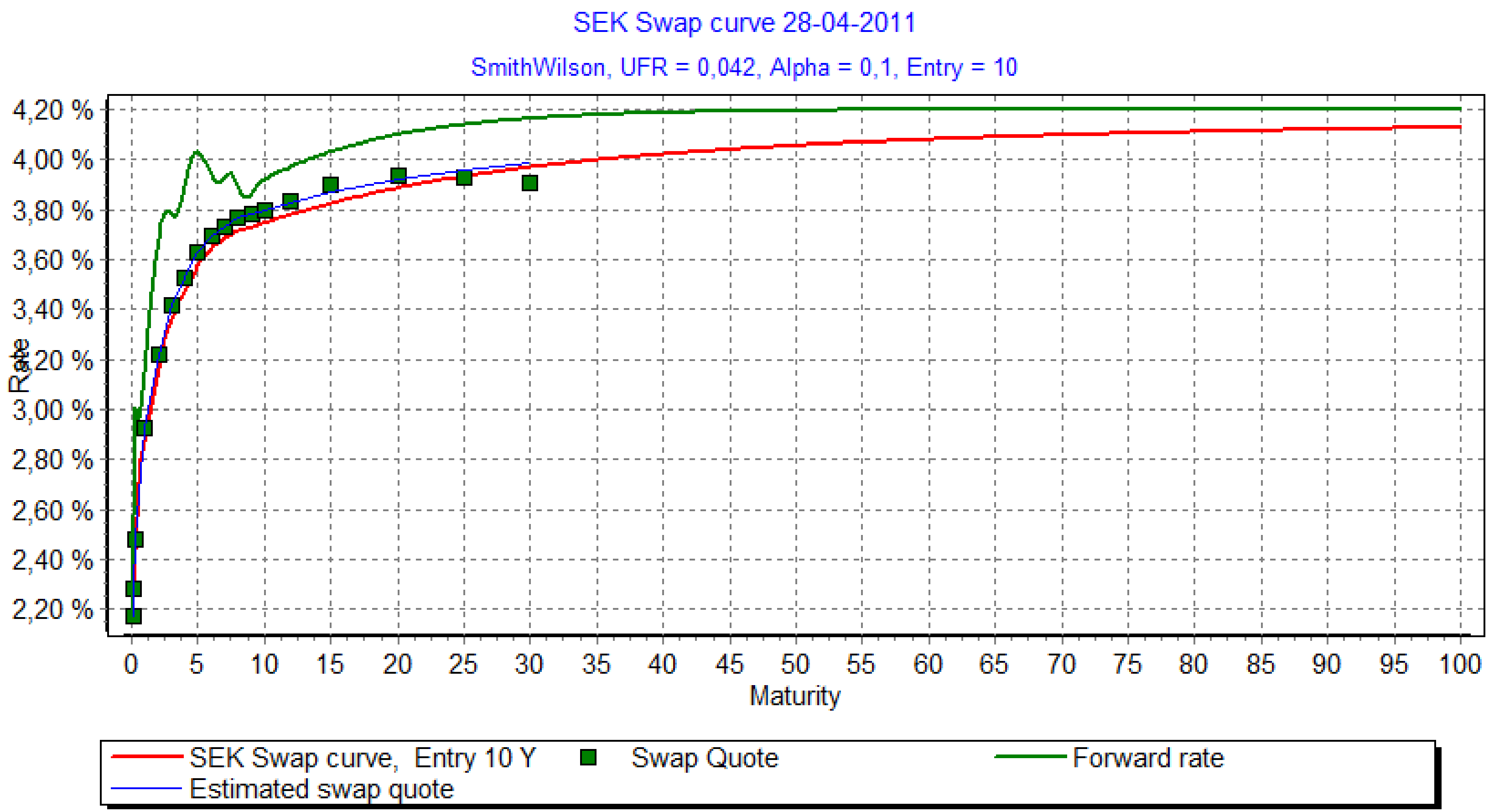
SEK swap curve



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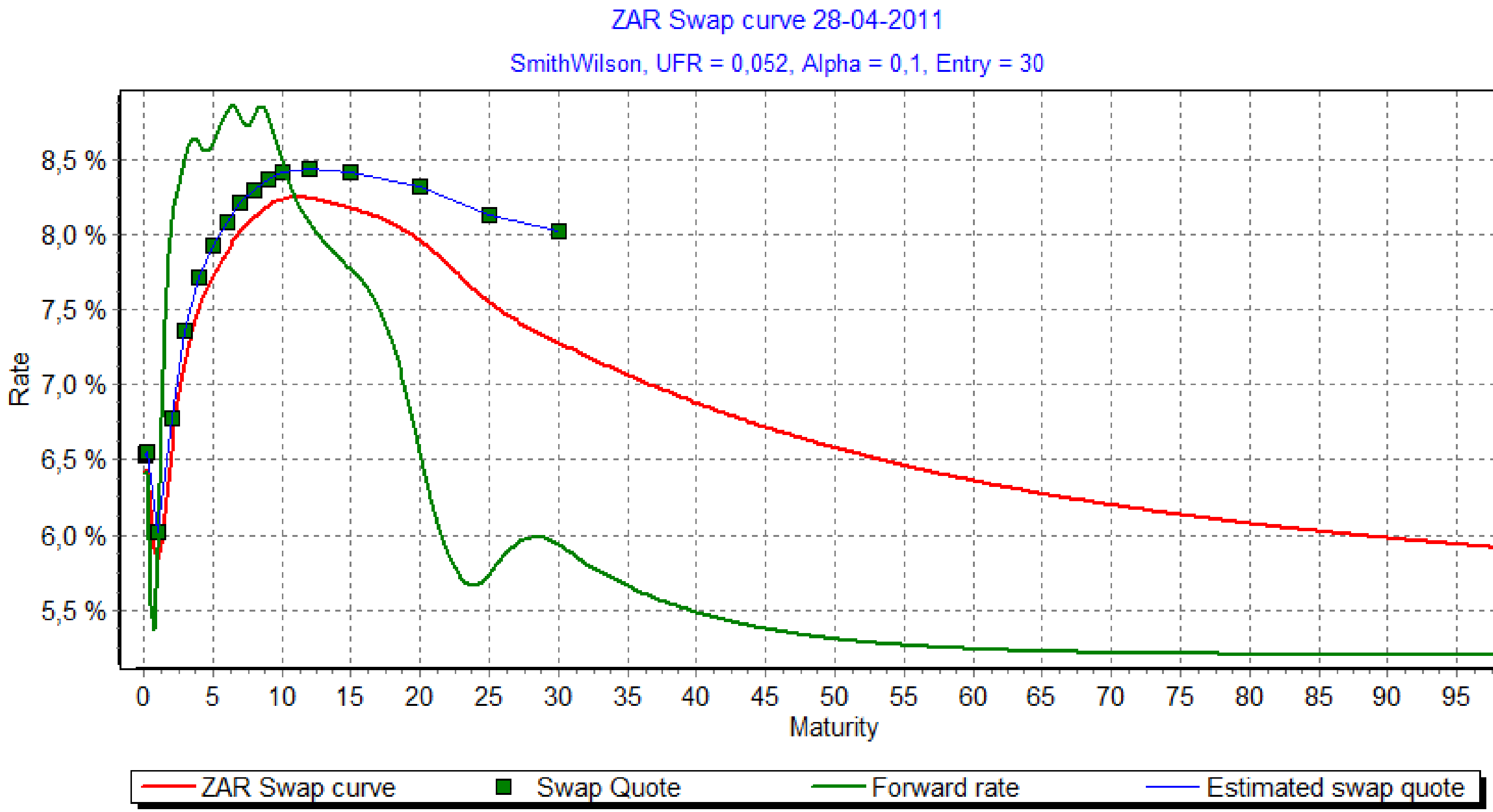
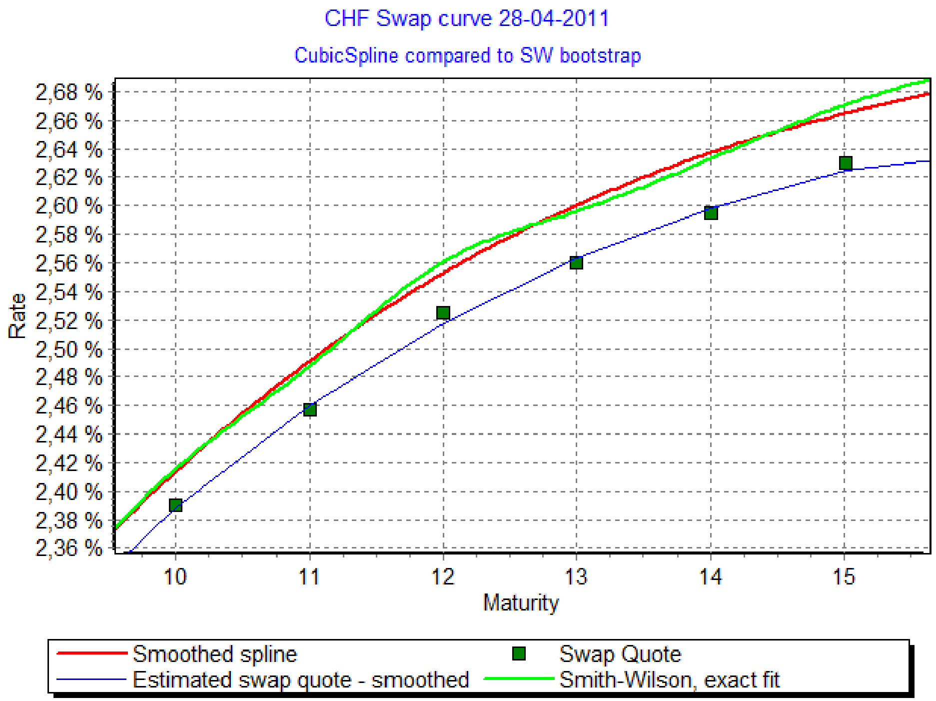
ZAR swap curve



Exact fit vs. smoothing /1

- Our implementation of the SW method provides an exact fit to the swap rates
- The chart shows part of the CHF swap curve estimated with the Scanrate smoothed cubic spline method compared to an SW curve with an exact fit
- To provide the exact fit the SW yield curve needs to be more ”bumpy” relative to the smoothed cubic spline.
- The smoothed cubic spline on the other hand does not provide an exact fit to the market rates
- Smoothing have little practical impact on the ability to match quotes
- The top 3 smoothing errors in this example are 1.8 bps (2Y), -1.2 bps (4Y) and 0.7 bps (12Y). Average absolute smoothing error is 0.36 bps.
- The SW method may result in unrealistic yield curves if the estimation sample contains outliers.
- For most swap markets the outlier problem will be minimal.



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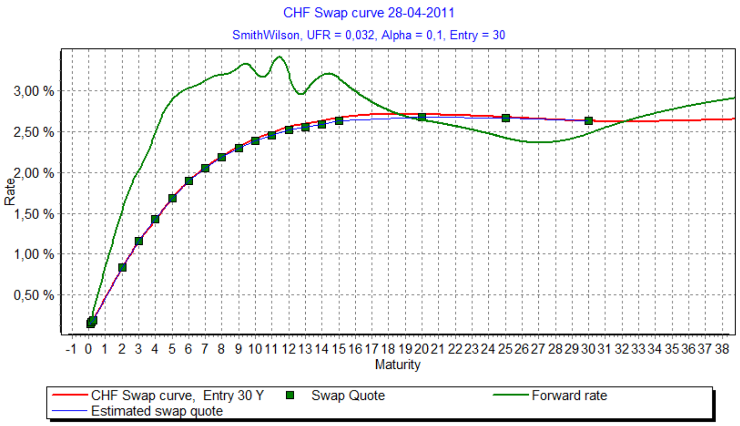
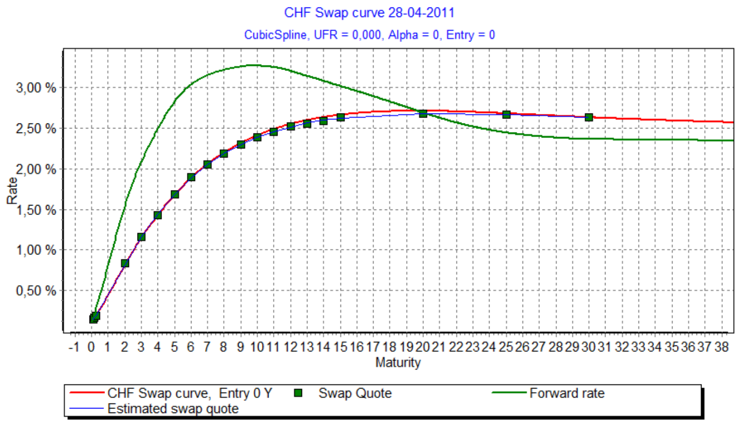
Exact fit vs. smoothing /2

- The non-smooth behavior of the SW spot rate curve is amplified when we look at forward rates.
- The smoothing spline curve provides a smooth forward curve as well.
- The smoothing spline procedure used by Scanrate is an extension of Tanggaard(1997). It requires non-linear optimization of the trade-off between pricing error and smoothness. The resulting curve parameters are easy to distribute and use, but the estimation procedure and thus replication by market participants will be more complicated than the SW method with an exact fit.
- Smoothing could be implemented for a SW type yield curve as well, but at the cost of simplicity.





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Exact fit vs. smoothing /3

- In practice the Solvency II yield curves will be estimated and distributed centrally – presumably from EIOPA
- To ease implementation the curves should be distributed as zero coupon yields at specific maturities, with an interpolation formula, which could be spline or SW. Similar to the QIS 5.
- Distributing as par swap quotes will increase implementation costs due to the extra cash flow generation step
- EIOPA should allow some room for smoothing in case of outliers
- Smoothing could be implemented as an integral part of the SW estimation procedure
- Smoothing will make it harder for market participants to replicate the centrally distributed yield curves to the last digit, but smoothing will be a minor issue relative to overall variation (and level!) of yields

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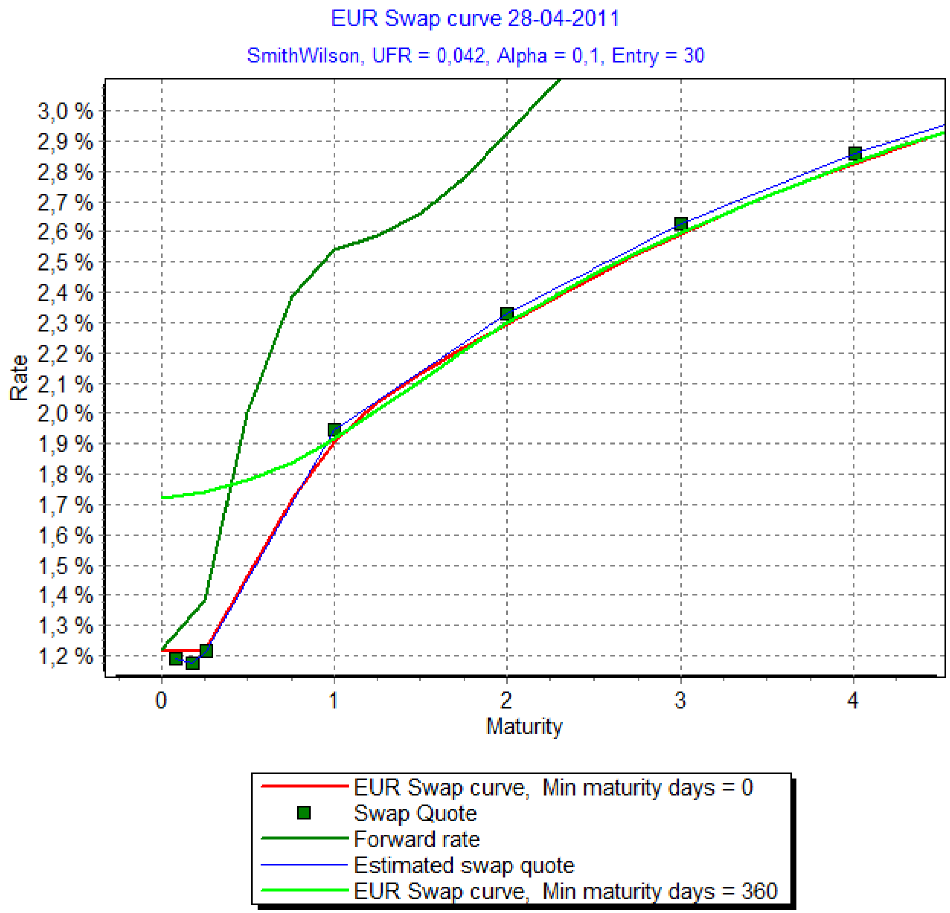
Short term extrapolation

- CFO Forum temporarily suggests 1Y swap as the first maturity due to differences in markets
- In general the SW yield curve extrapolates well in the short end (lime curve on chart), but the results may easily be improved upon by adding one or more short term rates (red curve)
- This will add realism and avoid unexpected short term volatility solely due to changes in longer term slopes



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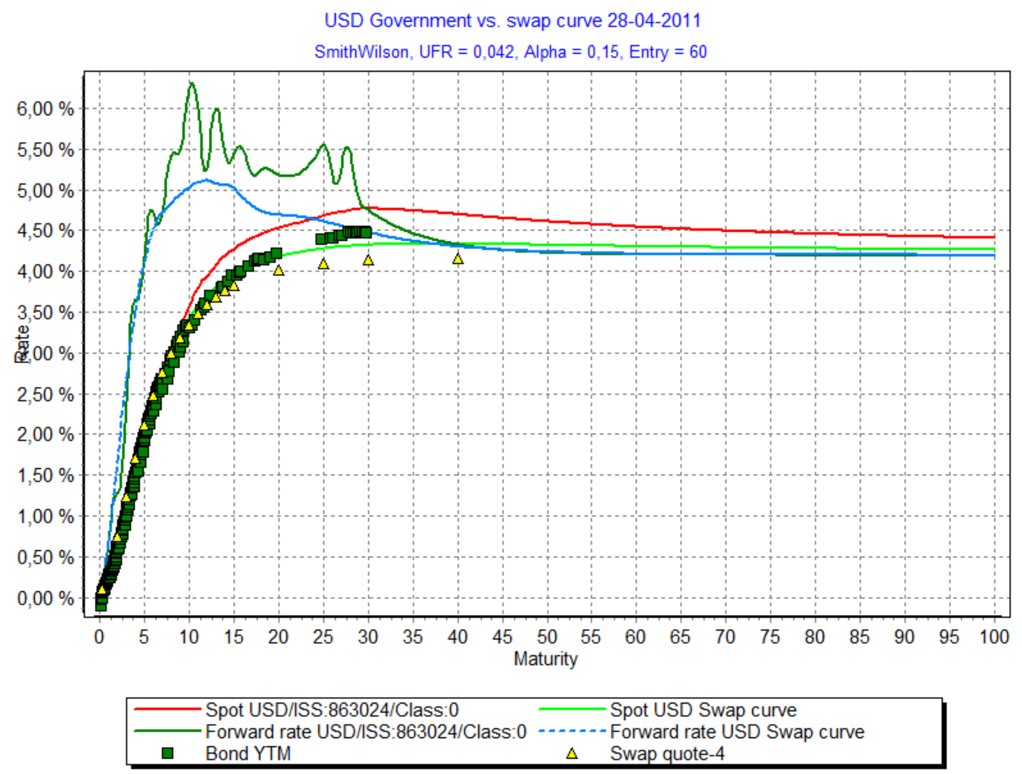
Use of government bond curves

- Fig: USD government and swap yield curves estimated with SW/UFR
- USD long term swap rates are currently below government rates
- The use of government curves implies regular update of relevant bond samples
- Special tax treatment in some markets
- SW forward rates are quite jagged for the US Gov curve. Smoothing or very careful sample selection will be needed for most bond markets -> the SW method should be replaced by smoothing spline methods
- No common EUR government curve
- In general swap curves are more relevant for hedging purposes



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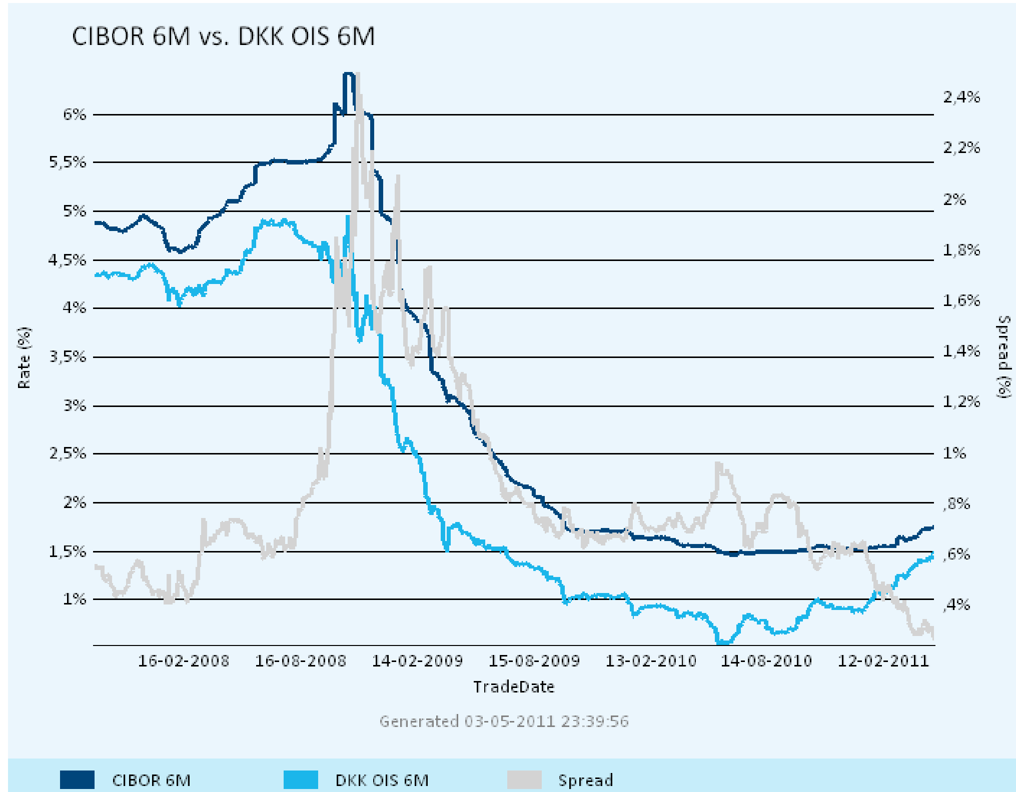
Choice of swap contract – LIBOR vs. OIS

- QIS 5 uses swap contracts against Libor/Euribor rates
- Libor rates reflects credit risk related to loss of prinicpal.
- No principal risk in the swap contract => fixed rates higher to compensate credit premium on floating leg
- Overnight index swaps are closer to the ”pure ”risk free curve
- OIS contracts are less liquid and shorter term according to the CRO Forum
- During crisis the choice of Libor based swap rates instead of OIS rates will typically reduce SCR
- *If the liquidity improves OIS rates would be the preferred choice for a risk free curve*



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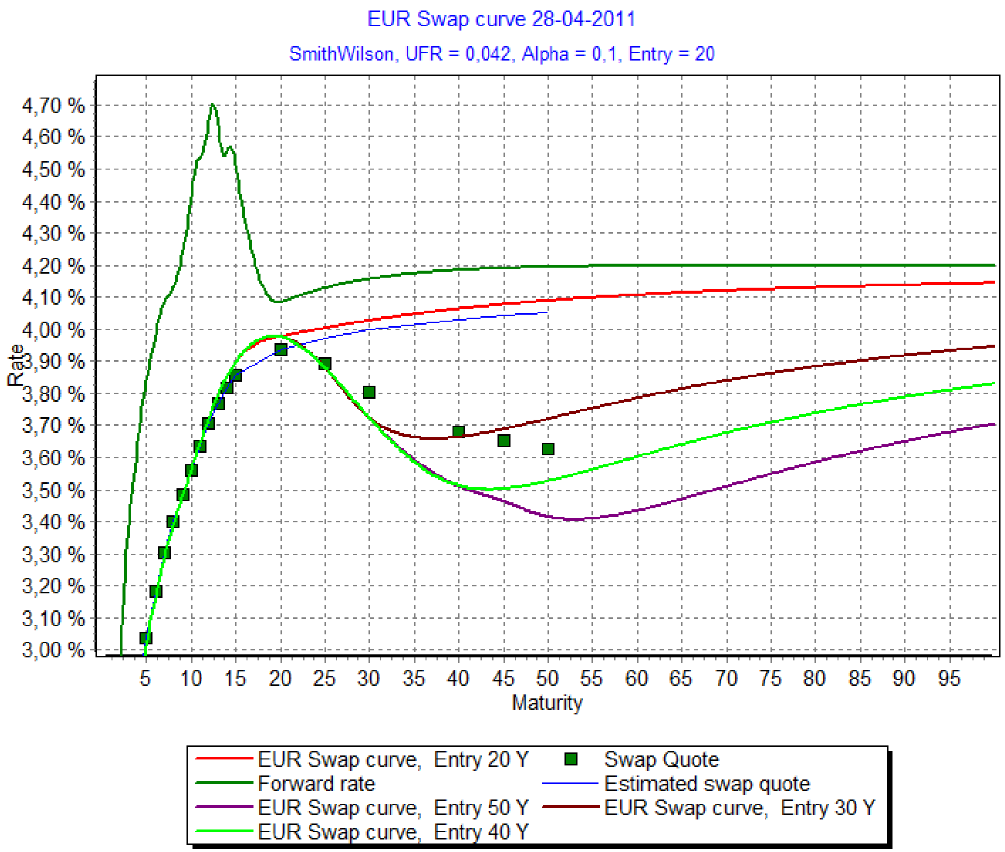
Entry point for long term extrapolation

- According to QIS 5 the extrapolation should start at last liquid maturity
- CFO/CRO Forum indicators for illiquidity: # of contributors, negative long slope, swap below gov, swap << UFR, long vol > short vol
- The forum sees lower long liquidity and higher vol during crisis 2008-2009,
- The CFO forum suggests lower entry points in crisis situations
- The entry point to extrapolation may have a very large impact on long term yields
- Different currencies could have different long term levels rates due to liquidity alone



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Basic risk free curve - conclusions

- The Smith-Wilson bootstrap method works quite well for the swap curves shown and it extrapolates to an exogenously given UFR.
- We have implemented the SW method using swap cash flows (fixed leg) instead of zero coupon yields as input. However, knowledge of market conventions for each swap market is needed to replicate the calibration
- SW forward rates show jagged behavior in several cases (ZAR, CHF). Primarily caused by the requirement of an exact math to market rates.
- The SII yield curves should be distributed as zero coupon yields. Some smoothing should be allowed for in the estimation procedure. Smoothing makes replication of the distributed curve from par rates a bit harder.
- To control the short end of the curve we suggest to add short term rate(s) below 1Y
- Government curves – especially EUR - are not practical for the risk free curve
- Overnight index swap contracts should be considered when liquidity increases
- Large SCR impact from changes in entry point to extrapolation

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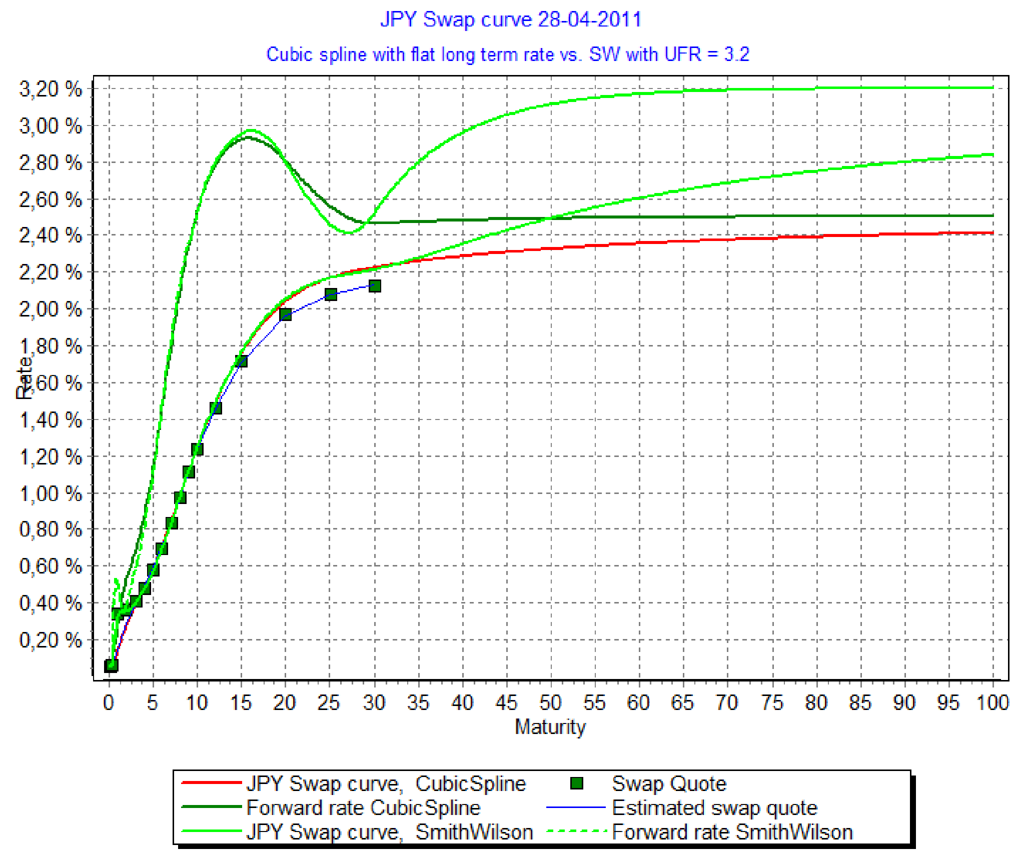
The unconditional forward rate

Extrapolation of long term yields

- Scanrate models the yield curve as a natural cubic spline smoothed and with a transformation to ensure a flat long term yield
- The long rate is thus determined endogenously by the last observed data point
- QIS 5 makes the UFR exogenous and determined by macroeconomic analysis
- Both models fit *observed* market data, but the long term yield could be quite different
- Which assumption is best ?
- How should we determine the long rate?



QIS 5 choice of UFR

- The QIS 5 UFR is determined as the average world wide real interest rate for the last 50 years, 2.2 per cent, plus the average inflation target for most central banks, 2.0 per cent.
- UFR equals 4.2 per cent for most currencies (EUR, USD, GBP, SEK, DKK, AUD, PLNB etc.)
- For low inflation currencies (CHF, JPY) the UFR is reduced to 3.2 per cent
- For high inflation countries like Turkey, Brazil, Mexico, South Africa UFR is increased to 5.2 per cent
- The UFR will be subject to regular revision, but should only be changed in case of fundamental changes in long term expectations
- Term premiums and convexity effects are not included in the forecast

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Growth of long term liabilities = UFR. Today!

- Consider a long term nominal liability L at future maturity T
- The present value at time t is P(t,T)*L
- Next year the present value will be P(t+1,T-1)*L
- The rate of increase in PV will be P(t+1,T-1)/ P(t,T)-1
- If the yield curve stays unchanged, i.e. P(t+1,T-1) = P(t,T-1) then the rate of increase in liabilities will equal f(t,T-1,T); the maturity T-1 forward rate at time t
- In QIS 5 this rate has been fixed at UFR for long term liabilities
- In stable yield conditions under Solvency II the present value of long term liabilities will thus increase with the UFR *every* year!
- The value of assets on the other hand will increase with market rates
- If UFR is high relative to the return on assets the institutions may not be able to meet future SCR
- Catch 22: If UFR is low institutions may not meet current SCR

UFR – interest rate (in)sensitivity

- Some Danish pension funds delta hedge very long term liabilities using 20-30 year swaps assuming a flat long term yield
- Long term liabilities are not cash flow matched, but from year to year the hedge will roll forward into new long term swap contracts.
- Errors in the judgment of long term forward rates will show up as gain/losses on the delta hedge
- The UFR method makes very long term liabilities less sensitive to changes in observed yields relative to a flat extrapolation
- What looks like a well hedged position from a flat extrapolation perspective may turn into a volatile position from a regulatory UFR perspective

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Choice of UFR - comments

- Under stable conditions long term liabilities increase with UFR while assets earn current market rates
- The choice of a constant UFR makes long term liabilities less sensitive to changes in long term yields
- Constant UFR discourages delta hedging of long term liabilities as this will lead to a volatile solvency level
- To meet SCR every year under stable conditions the UFR should equal a rate which could be earned in the market today
- To encourage delta hedging of liabilities the UFR should move with long term swap rates
- A suggestion would be to set UFR = long term rate at entry point
- If the UFR moves with long term market rates the non-hedged institutions would experience volatile SC levels => pressure to hedge or to reduce the use of life long guarantees

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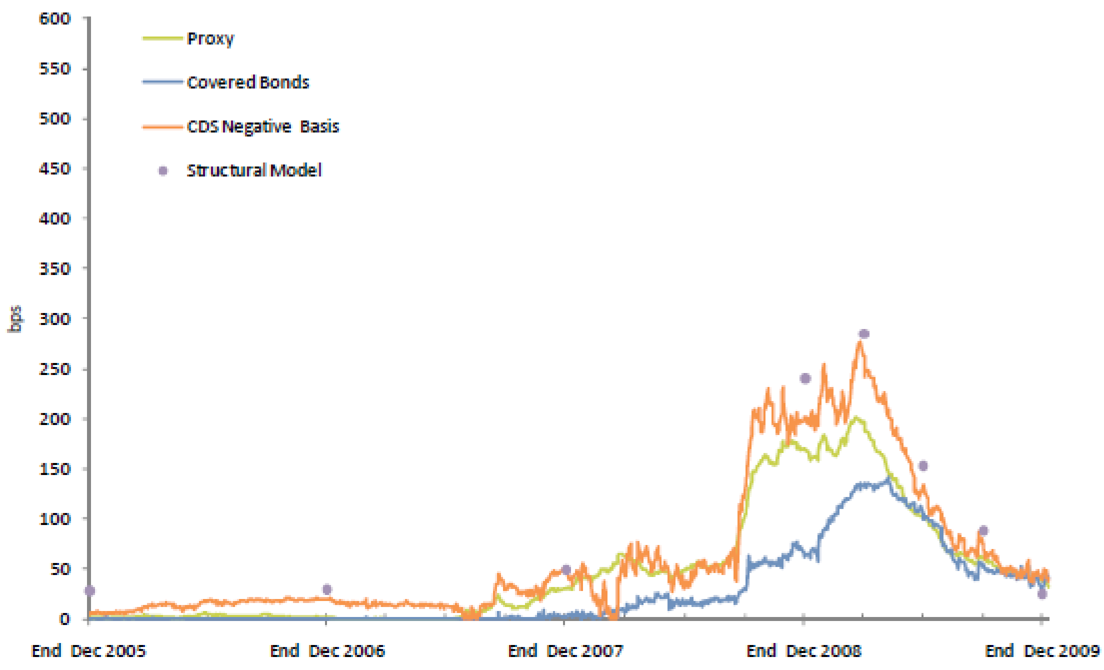
Illiquidity premium

The illiquidity premium

- The illiquidity premium (ILP) is added to the risk free yield curve. Is meant to reflect illiquidity, but not credit risk
- The higher discount rate for liabilities reflects that long term investors are able to receive consistently higher returns from investment in illiquid assets
- The illiquidity premium typically increase during crisis. Reduction in asset values are thus counteracted by a reduction in liabilities

Candidates for spread

EUR – Derivation of liquidity premium



Illiquidity premium – QIS 5

- Added from time zero to the last liquid market rate
- Average corporate bond spreads are found from indices (e.g. iBoxx)
- A constant fraction x of the spread is judged to be illiquidity, the remainder (1-x) is judged to be credit
- x is kept constant at 50%
- y = 40bps is subtracted to allow for average credit loss
- ILP = max(0,x*(spread-y)), x = 50%, y = 40 bps
- 10 bps subtracted to allow for swap to risk free spread
- Added to the *spot* rate, zero after entry point

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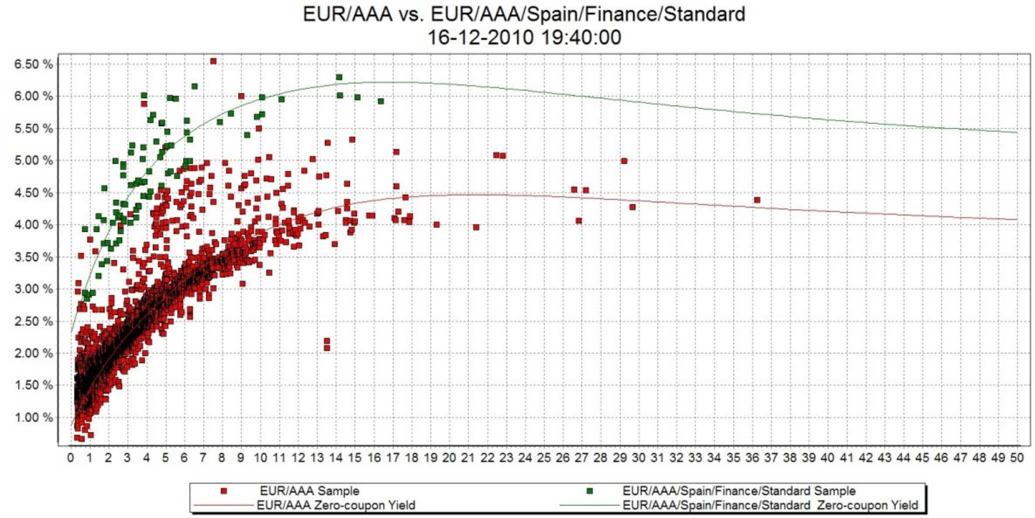
Corporate bonds spreads EUR AAA

- Bond spreads are highly dispersed and depend on the sample chosen – even within the same rating class
- The figure illustrates domicile effects within EUR AAA – Spanish financial institutions relative to full EUR AAA sample



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Illiquidity premium - comments

- ILP similar to QIS 5 would have had a huge influence on SCR during the 2008 crisis
- The ILP is dependent on the composition, credit quality and maturity structure of the bond indices used
- Deriving the ILP from a specific basket of assets (covered bonds, corporate bond indices) induces a need for hedging against a decrease of the ILP in a subset of the market which is by definition illiquid! The end result could be distorted pricing for the basket of assets
- The ILP is currency specific, but institutions invest in global portfolios
- The primary purpose of the ILP is to reduce SCR during financial crisis – maybe the ILP should be kept as an explicit parameter to be changed by regulatory authorities on a discretionary basis?
- For practical reasons the ILP should be added to the forward rate and leveled out. Jumps in spot rate is a showstopper for modeling

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